Digital Signal Processing
TE Sem V

Module 1: Discrete Fourier Transform and Fast Fourier

Topic	Suggested Book	Remarks
Definition and Properties of DFT, IDFT, circular convolution of sequences using DFT and IDFT, Relation between Z-transform and DFT	Tarun Kumar Rawat, " Digital Signal Processing", Oxford University Press, 2015 Pages 393-434, 447	Theory/Properties sums
Filtering of long data sequences: Overlap Save and Overlap Add Method	Pages 437-442	Theory, numericals
Computation of DFT	Pages 393-434	4 point/8 point DFT sums
Fast Fourier transforms(FFT), Radix-2 decimation in time and decimation in frequency FFT algorithms, inverse FFT, and **Introduction to composite FFT**	Pages 471-498 ProakisJ., Manolakis D., "*Digital Signal Processing*", 4th Edition, Pearson Education Pages 514-518	Theory/ **No numerical on Composite FFT**

Module 2: IIR Digital Filters (10 Hrs,25 Marks)

Topic	Suggested Book	Remarks
Types of IIR Filters (Low Pass, High Pass, Band Pass, Band stop and All Pass) Analog filter approximations: Butterworth, Chebyshev I and II	Tarun Kumar Rawat, " Digital Signal Processing", Oxford University Press, 2015 Pages 350-358, 749-776	Numericals Expected (10) on LPF **(Butterwoth)** Derivations (Butterworth LPF, (No numerical Chebyshev I and II)
Mapping of S-plane to Z-plane, impulse invariance method, bilinear transformation method, Design of IIR digital filters from analog filters with examples	724-735 738-747	Derivations Numericals on Filter Design(10)
Analog and digital frequency transformations with design examples	776-791	Numericals Ex Pected (5)

Filter design with Matlab examples given in the Ifeachor for experiments/assignments
Proakis J., Manolakis D., "Digital Signal Processing", 4th Edition, Pearson Education can be used as reference book

Module 3: FIR Digital Filters (10 Hrs,25 Marks)

Topic	Suggested Book	Remarks
Characteristics of FIR digital filters, Minimum Phase, Maximum Phase, Mixed Phase and Linear Phase Filters Frequency response, location of the zeros of linear phase FIR filters	Emmanuel C. Ifeachor, Barrie W. Jervis, “Digital Signal Processing”, A Practical Approach by, Pearson Education Pages 342-351	Theory (10)
Design of FIR filters using window techniques (Rectangular, Hamming, Hanning, Blackmann, Barlet) Design of FIR filters using Frequency Sampling technique Comparison of IIR and FIR filters	Pages 352-366, 380-398	Numericals on Filter Design, (No numerical on Frequency sampling) (windowing)(10) Theory-5

Filter design with Matlab examples given in the book for experiments/assignments

Module 4: Finite Word Length Effects in Digital Filters
6 Hrs, 15 Marks

Topic	Suggested Book	Remarks
Quantization, truncation and rounding, Effects due to truncation and rounding, Input quantization error, Product quantization error**, Co-efficient quantization error,** Zero input limit cycle oscillations, Overflow limit cycle oscillations, Scaling	Tarun Kumar Rawat, " Digital Signal Processing", Oxford University Press, 2015 Pages 812-882	Theory /Numerical on coefficient quantization error (10)
Quantization in Floating Point realization of IIR digital filters Finite word length effects in FIR digital filters		Theory

Module 5: Multirate DSP and Filter Banks (6 Hrs, 15 Marks)

Topic	Suggested Book	Remarks
Introduction and concept of Multirate Processing, Block Diagram of Decimator and Interpolator, Decimation and Interpolation by Integer numbers Multistage Approach to Sampling rate converters	Emmanuel C. Ifeachor, Barrie W. Jervis, “Digital Signal Processing”, A Practical Approach by, Pearson Education Pages 579-590	Theory, **No Numericals on Multistage**
Sample rate conversion using Polyphase filter structure, Type I and Type II Polyphase Decomposition	Pages 612-617	Theory/ Simple Numericals-5

Practical Rate Converters not included, applications can be given as topics for presentation

Module 6: DSP Processors and Applications (6 Hrs, 15 Marks)

Topic	Suggested Book	Remarks
Computer architecture for signal processing, Harvard Architecture, Pipelining, multiplier and accumulator(MAC), Special Instructions, Replication, On-chip memory, Extended Parallelism	Emmanuel C. Ifeachor, Barrie W. Jervis, "Digital Signal Processing", A Practical Approach by, Pearson Education Pages 727-746	Theory, No Numericals For 3 Marks
Introduction to General Purpose and Special Purpose DSP processors, fixed point and floating point DSP processor, General purpose digital signal processors, Selecting digital signal processors, Special purpose DSP hardware (Just introdction)	Pages 746-761 787-788	Theory For 6 Marks
Applications of DSP: Radar Signal Processing and Speech Processing (Biomedical, Image Processing)	Pages 672-673 Pages 16-19	Theory, Block diagram representation For 6 Marks

Implementation of algorithms(Theory) in not expected

Books

- Text Books and References:
  - Emmanuel C. Ifeachor, Barrie W. Jervis, “Digital Signal Processing”, A Practical Approach by, Pearson Education
  - Tarun Kumar Rawat, “ Digital Signal Processing”, Oxford University Press, 2015
  - Proakis J., Manolakis D., "Digital Signal Processing", 4th Edition, Pearson Education

Sample Mini projects

- Audio Signal data and demonstration of effect of decimation and interpolation on the recorded signal
- Demonstration of filters on speech signal data
- Biomedical signal like ECG, EEG
- Miniproject scope should not be too vast, select topic such that it can be completed in 2-3 weeks

Module 1: Discrete Fourier Transform and Fast Fourier

Topic	Suggested Book	Remarks
Definition and Properties of DFT, IDFT, circular convolution of sequences using DFT and IDFT, Relation between Z-transform and DFT	Tarun Kumar Rawat, " Digital Signal Processing", Oxford University Press, 2015 Pages 393-434, 447	Theory/Properties sums
Filtering of long data sequences: Overlap Save and Overlap Add Method	Pages 437-442	Theory, numericals
Computation of DFT	Pages 393-434	4 point/8 point DFT sums
Fast Fourier transforms(FFT), Radix-2 decimation in time and decimation in frequency FFT algorithms, inverse FFT, and **Introduction to composite FFT**	Pages 471-498 ProakisJ., Manolakis D., "*Digital Signal Processing*", 4th Edition, Pearson Education Pages 514-518	Theory/ **No numerical on Composite FFT**

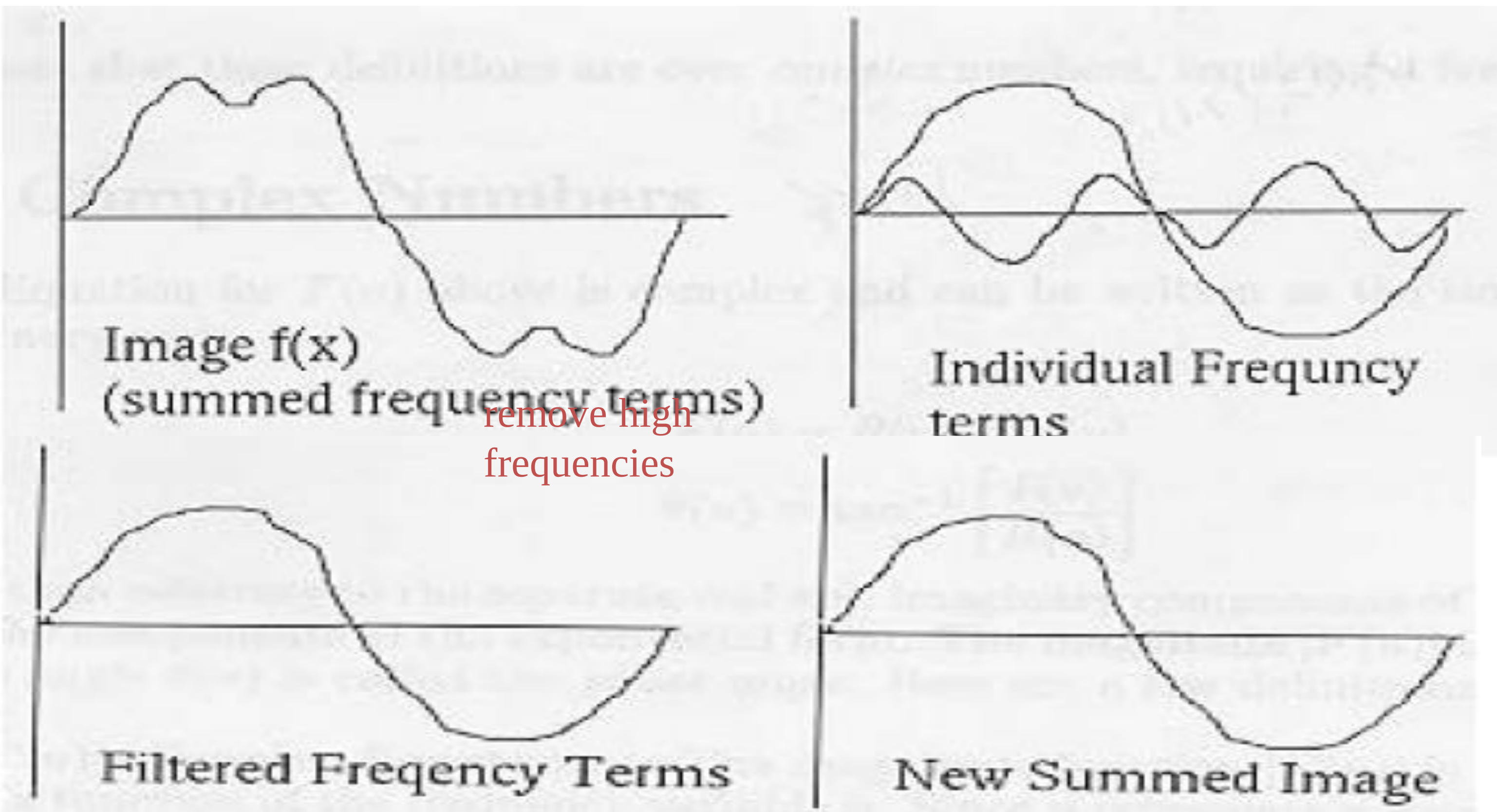
Why is FT Useful?

- Easier to remove undesirable frequencies in the **frequency** domain.

- Faster to perform certain operations in the **frequency** domain than in the **time** domain.

Why to use FT?

Removing undesirable frequencies



DTFT and DFT

Recall the DTFT:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}.$$

DTFT is not suitable for DSP applications because

- In DSP, we are able to compute the spectrum only at specific discrete values of ω,
- Any signal in any DSP application can be measured only in a finite number of points.

DFT and IDFT

Sample the spectrum $X(\omega)$ in frequency so that

$$
\begin{aligned}
X(k) &= X(k\Delta\omega), \quad \Delta\omega = \frac{2\pi}{N} \quad \Longrightarrow \\
X(k) &= \sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{kn}{N}} \quad \text{DFT.}
\end{aligned}
$$

The **inverse DFT** is given by:

$$
x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi\frac{kn}{N}}.
$$

DFT pair

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{kn}{N}} \quad \text{analysis}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi\frac{kn}{N}} \quad \text{synthesis.}$$

- **Given the signal:**

$$x[0] = 1,\ x[1] = 2,\ x[2] = 2,\ x[3] = 1,\ x[n] = 0 \text{ otherwise} \rightarrow \mathbf{x} = [1,2,2,1]$$

$$X_k = \sum_{n=0}^{3} x[n]e^{-j2\pi kn/4},\ \ k = 0, 1, 2, 3$$

$$= x[0]e^{-j2\pi k(0)/4} + x[1]e^{-j2\pi k(1)/4} + x[2]e^{-j2\pi k(2)/4} + x[3]e^{-j2\pi k(3)/4}$$

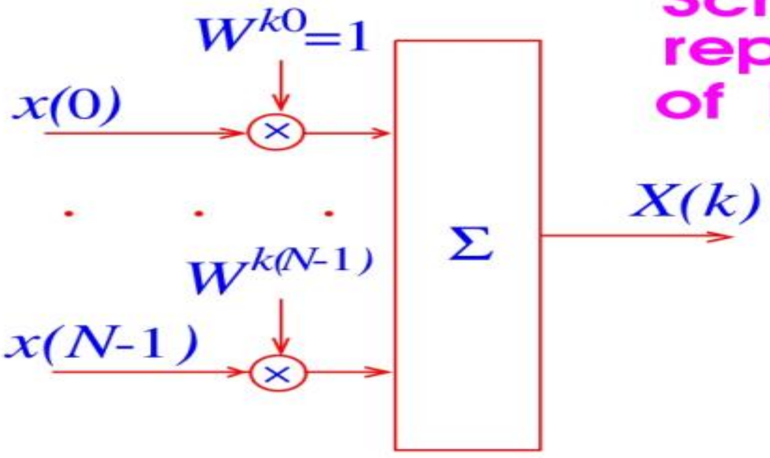
$$x[0]=1,\ x[1]=2,\ x[2]=2,\ x[3]=1,\ x[n]=0 \text{ otherwise} \rightarrow \mathbf{x}=[1,2,2,1]$$

$$\begin{aligned}
X_k &= \sum_{n=0}^{3} x[n] e^{-j2\pi kn/4},\ \ k=0,1,2,3 \\
&= x[0]e^{-j2\pi k(0)/4} + x[1]e^{-j2\pi k(1)/4} + x[2]e^{-j2\pi k(2)/4} + x[3]e^{-j2\pi k(3)/4} \\
&= 1 + 2e^{-j\pi k/2} + 2e^{-j\pi k} + 1e^{-j\pi 3k/2},\ \ k=0,1,2,3 \\
&= \left[1 + 2\cos\left(\frac{-\pi k}{2}\right) + 2\cos(-\pi k) + \cos\left(\frac{-3\pi k}{2}\right)\right] \\
&\quad + j\left[-2\sin\left(\frac{\pi k}{2}\right) - 2\sin(\pi k) - \sin\left(\frac{3\pi k}{2}\right)\right]
\end{aligned}$$

DFT:twiddle factor

Alternative formulation:

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n) W^{kn} \quad \longleftarrow \quad W = e^{-j\frac{2\pi}{N}} \\
x(n) &= \frac{1}{N} \sum_{k=0}^{N-1} X(k) W^{-kn}.
\end{aligned}
$$



Schematic representation of DFT

Periodicity of DFT Spectrum

$$
\begin{aligned}
X(k+N) &= \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{(k+N)n}{N}} \\
&= \left(\sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}} \right) e^{-j2\pi n} \\
&= X(k) e^{-j2\pi n} = X(k) \Longrightarrow
\end{aligned}
$$

the DFT spectrum is periodic with period N (which is expected, since the DTFT spectrum is periodic as well, but with period 2π).

DFT computation:-Method 1

- **Given the signal:**

$$x[0]=1,\ x[1]=2,\ x[2]=2,\ x[3]=1,\ x[n]=0 \text{ otherwise} \rightarrow \mathbf{x}=[1,2,2,1]$$

$$X_k=\sum_{n=0}^{3} x[n]e^{-j2\pi kn/4},\quad k=0,1,2,3$$

$$=x[0]e^{-j2\pi k(0)/4}+x[1]e^{-j2\pi k(1)/4}+x[2]e^{-j2\pi k(2)/4}+x[3]e^{-j2\pi k(3)/4}$$

$$=1+2e^{-j\pi k/2}+2e^{-j\pi k}+1e^{-j\pi 3k/2},\quad k=0,1,2,3$$

$$=\left[1+2\cos(\frac{-\pi k}{2})+2\cos(-\pi k)+\cos(\frac{-3\pi k}{2})\right]$$

$$+j\left[-2\sin(\frac{\pi k}{2})-2\sin(\pi k)-\sin(\frac{3\pi k}{2})\right]$$

$$X_k=\begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

Inverse DFT computation

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi\frac{kn}{N}}.$$

Inverse DFT computation

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi\frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[n] = \frac{1}{4}\left[X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}\right]$$

Inverse DFT computation

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[n] = \frac{1}{4}\left[X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}\right]$$

$$x[0] = \frac{1}{4}\left[X_0 + X_1 + X_2 + X_3\right] = \frac{1}{4}\left[6-1-j-1+j\right] = \frac{4}{4} = 1$$

Inverse DFT computation

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[n] = \frac{1}{4}\left[X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}\right]$$

$$x[1] = \frac{1}{4}\left[X_0 + X_1 e^{j\pi/2} + X_2 e^{j\pi} + X_3 e^{j3\pi/2}\right];$$

Inverse DFT computation

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[1] = \frac{1}{4}\left[X_0 + X_1 e^{j\pi/2} + X_2 e^{j\pi} + X_3 e^{j3\pi/2}\right];$$

$$= \frac{1}{4}\left[6 + (-1-j)j + (0)(-1) + (-1+j)(-j)\right]$$

Inverse DFT computation

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi\frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[1] = \frac{1}{4}\left[X_0 + X_1 e^{j\pi/2} + X_2 e^{j\pi} + X_3 e^{j3\pi/2}\right];$$

$$= \frac{1}{4}\left[6 + (-1-j)j + (0)(-1) + (-1+j)(-j)\right]$$

$$= \frac{1}{4}\left[6 - j + 1 + j + 1\right] = 8/4 = 2$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}}.$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[n] = \frac{1}{4}\left[X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}\right]$$

$$x[2] = \frac{1}{4}\left[X_0 + X_1 e^{j\pi} + X_2 e^{j2\pi} + X_3 e^{j3\pi}\right];$$

$$x[3] = \frac{1}{4}\left[X_0 + X_1 e^{j3\pi/2} + X_2 e^{j3\pi} + X_3 e^{j18\pi/4}\right];$$

$$x[n] = \frac{1}{4}\left[X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}\right]$$

$$X_k = \begin{cases} 6 & k=0 \\ -1-j & k=1 \\ 0 & k=2 \\ -1+j & k=3 \end{cases}$$

$$x[2] = \frac{1}{4}\left[X_0 + X_1 e^{j\pi} + X_2 e^{j2\pi} + X_3 e^{j3\pi}\right] = \frac{1}{4}[6 + (-1-j)(-1) + (0) + (-1+j)(-1)]$$

$$= \frac{1}{4}[6 + 1 + j + 1 - j] = 8/4 = 2$$

$$x[3] = \frac{1}{4}\left[X_0 + X_1 e^{j3\pi/2} + X_2 e^{j3\pi} + X_3 e^{j18\pi/4}\right] = \frac{1}{4}[6 + (-1-j)(-j) + (0) + (-1+j)(j)]$$

$$= \frac{1}{4}[6 + j - 1 - j - 1] = 4/4 = 1$$

summary

- The computation procedure for X(K) and x(n) is quite complex
- The DFT requires NxN complex multiplications and N(N-1) complex additions. Without the twiddle factor, the computational complexity of DFT is $O(N^2)$.
- With twiddle factors, the computational complexity is Nlog2N.

What are twiddle factors?

- Twiddle are a set of values that is used to speed up DFT and IDFT calculations.

Twiddle factors are mathematically represented as:

$$W_N = e^{-j2\pi/N}$$

Rewriting the equations for calculating DFT and IDFT using twiddle factors we get:

DFT: $\sum_{n=0}^{N-1} x(n) W_N^{nk}$

IDFT: $\frac{1}{N} \sum_{n=0}^{N-1} x(k) W_N^{-nk}$

Why do we use twiddle factors?

- We use the twiddle factor to reduce the computational complexity of calculating DFT and IDFT.
- The twiddle factor is a rotating vector quantity. All that means is that for a given N-point DFT or IDFT calculation, it is observed that the values of the twiddle factor repeat at every N cycles.

Twiddle factor matrix computation

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & \cdots & W_N^{N-1} \\ . & W_N^2 & W_N^4 & \cdots & \cdots & W_N^{2(N-1)} \\ . & & & & & \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

It is denoted as W_N and defined as $W_N = e^{-j2\pi/N}$. Its magnitude is always maintained at unity. Phase of $W_N = -2\pi/N$. It is a vector on unit circle and is used for computational convenience. Mathematically, it can be shown as –

$$W_8^0 = W_8^8 = W_8^{16} = \ldots = \ldots = W_8^{32} = \ldots = 1 = 1\angle 0$$

$$= \begin{bmatrix} W_4^{0} & W_4^{0} & W_4^{0} & W_4^{0} \\ W_4^{0} & W_4^{1} & W_4^{2} & W_4^{3} \\ W_4^{0} & W_4^{2} & W_4^{4} & W_4^{6} \\ W_4^{0} & W_4^{3} & W_4^{6} & W_4^{9} \end{bmatrix}$$

$$W_4^{kn} = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

$$W_4^{kn} = \begin{array}{c|cccc} & n=0 & n=1 & n=2 & n=3 \\ k=0 & W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ k=1 & W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ k=2 & W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ k=3 & W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{array}$$

$$W_4^0 = e^{-j\frac{\pi}{2}\times 0} = e^0 = 1$$

$$W_4^1 = e^{-j\frac{\pi}{2}\times 1} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$W_4^2 = e^{-j\frac{\pi}{2}\times 2} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

Cyclic and periodicity property of Twiddle Factor

$$W_4^3 = e^{-j\frac{\pi}{2}\times 3} = e^{-j\frac{3\pi}{2}} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = +j$$

$$W_4^0 = W_4^4 = 1$$
$$W_4^1 = W_4^5 = W_4^9 = -j$$
$$W_4^2 = W_4^6 = W_4^{10} = -1$$
$$W_4^3 = W_4^7 = W_4^{11} = +j$$

$$W_4^{kn} = \begin{array}{c c} & \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} & \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$W_N^{k+N} = W_N^k$$
$$W_N^{k+\frac{N}{2}} = -W_N^k$$

Cyclic property of twiddle factors

For a 4-point DFT

Let's derive the twiddle factor values for a 4-point DFT

For n=0 and k=0,

$$W_N^{nk} = W_4^0 = e^0 = 1$$

(From Euler's formula: $e^{i\theta} = Cos\theta + iSin\theta$)

$$= \begin{bmatrix} W_4^{\,0} & W_4^{\,0} & W_4^{\,0} & W_4^{\,0} \\ W_4^{\,0} & W_4^{\,1} & W_4^{\,2} & W_4^{\,3} \\ W_4^{\,0} & W_4^{\,2} & W_4^{\,4} & W_4^{\,6} \\ W_4^{\,0} & W_4^{\,3} & W_4^{\,6} & W_4^{\,9} \end{bmatrix}$$

Similarly calculating for the remaining values we get the series below:

$W_4^0 = 1$

$W_4^1 = -j$

$W_4^2 = -1$

$W_4^3 = j$

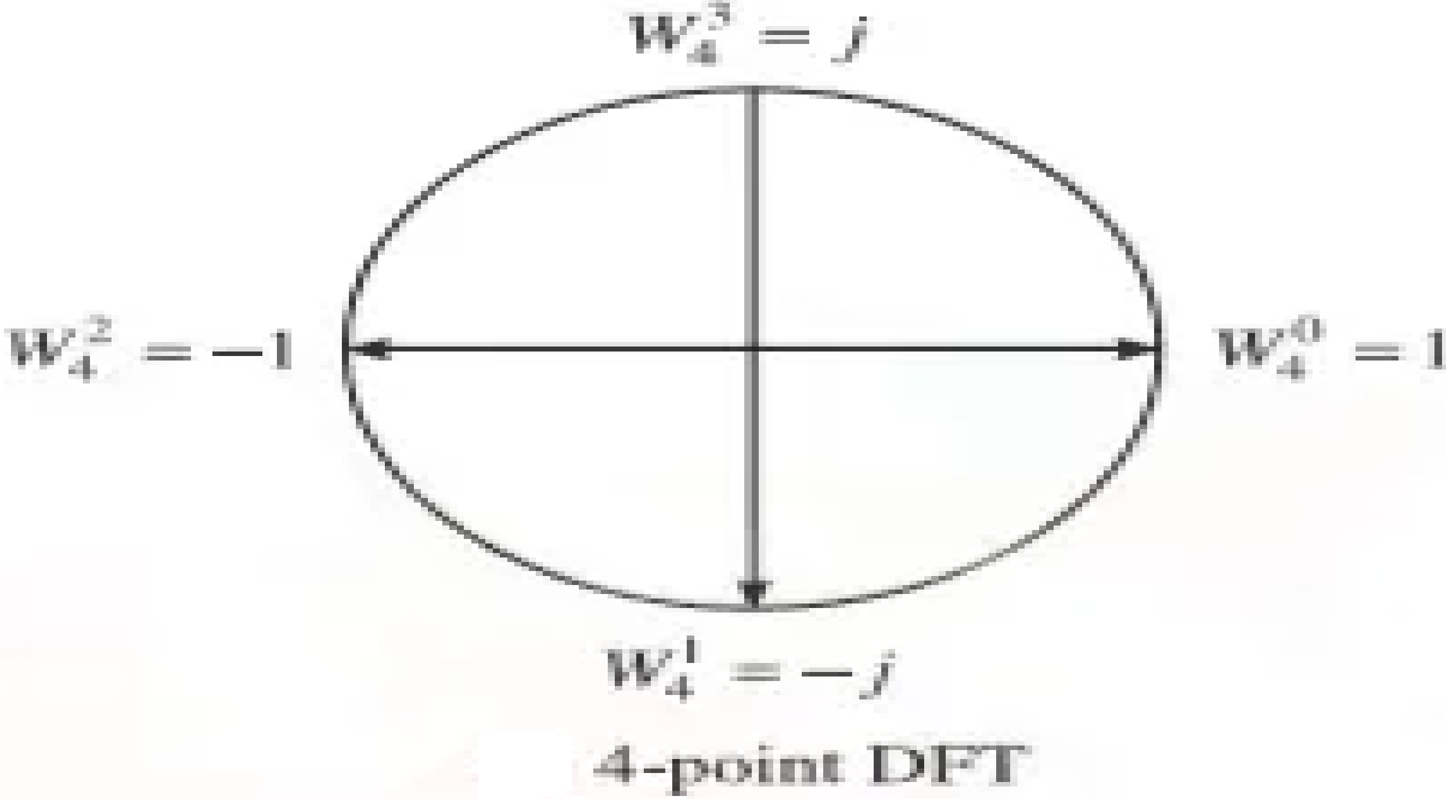
$W_4^4 = 1$

$$W_N^{\,k+N} = W_N^{\,k}$$

$$W_N^{\,k+\frac{N}{2}} = -W_N^{\,k}$$

As you can see, the value starts repeating at the 4th instant.

Twiddle factor on unit circle



DFT computation:Method 2

DFT: $\sum_{n=0}^{N-1} x(n) W_N^{nk}$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

DFT computation:Method 2

DFT: $\sum_{n=0}^{N-1} x(n)W_N^{nk}$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ 2j-2 \\ -2 \\ -2j-2 \end{bmatrix}$$

Cyclic property of twiddle factors

$W_8^0 = 1$

$W_8^1 = 0.707\text{-}0.707j$

$W_8^2 = \text{-}j$

$W_8^3 = \text{-}0.707\text{-}0.707j$

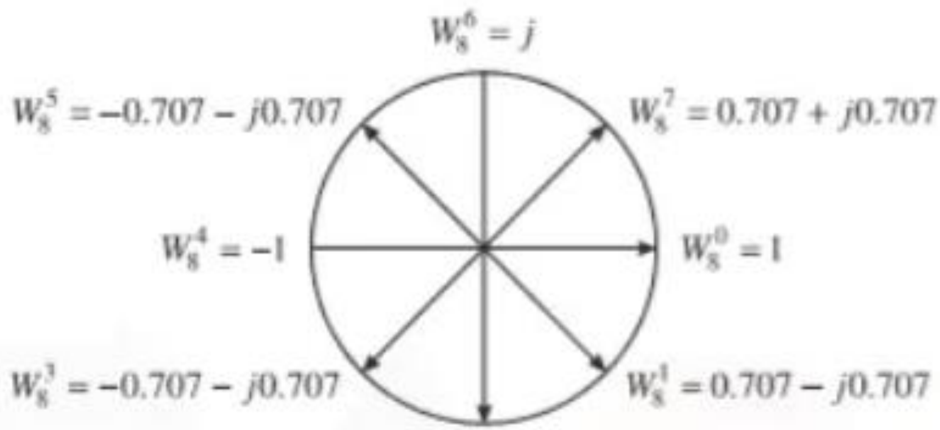
$W_8^4 = \text{-}1$

$W_8^5 = \text{-}0.707\text{+}0.707j$

$W_8^6 = j$

$W_8^7 = 0.707\text{+}0.707j$

$W_8^8 = 1$



8-point DFT

Linear convolution

- https://www.slideserve.com/chance/dft-properties

DFT properties

http://eeweb.poly.edu/iselesni/EL713/zoom/dftprop.pdf

$$X(1) = 1 + \left(\cos\frac{2\pi}{4} - j\sin\frac{2\pi}{4}\right) + \left(\cos\frac{4\pi}{4} - j\sin\frac{4\pi}{4}\right) + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right)$$

$$X(1) = 1 + (0 - j) + (-1 - 0) + (0 + j)$$

$$X(1) = 1 - j - 1 + j = 0$$

$$X(2) = \sum_{n=0}^{3} e^{-j2\pi \times 2n/4} = \sum_{n=0}^{3} e^{-j\pi n}$$

$$X(2) = e^{0} + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi}$$

$$X(2) = 1 + (\cos \pi - j \sin \pi) + (\cos 2\pi - j \sin 2\pi) + (\cos 3\pi - j \sin 3\pi)$$

$$X(2) = 1 + (-1 - 0) + (1 - 0) + (-1 - 0)$$

$$= 1 - 1 + 1 - 1 = 0$$

$$X(3) = \sum_{n=0}^{3} e^{-j2\pi \times 3n/4} = \sum_{n=0}^{3} e^{-j6\pi n/4}$$

$$X(3) = e^{0} + e^{-j6\pi/4} + e^{-j3\pi} + e^{-j9\pi/2}$$

$$X(3) = 1 + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right) + (\cos 3\pi - j \sin 3\pi) + \left(\cos\frac{9\pi}{2} - j\sin\frac{9\pi}{2}\right)$$

$$X(3) = 1 + (0 + j) + (-1 - 0) + (0 - j) = 1 + j - 1 - j = 0$$

$$X(k) = \{4, 0, 0, 0\}$$

Circular Convolution:-Time Domain